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Some Exact Bianchi Type-V Perfect Fluid Cosmological Models with Heat Flow and Decaying Vacuum Energy Density Λ: Expressions for Some Observable Quantities

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Abstract In this paper we have obtained some new exact solutions of Einstein's field equations in a spatially homogeneous and anisotropic Bianchi type-V space-time with perfect fluid distribution along with heat-conduction and decaying vacuum energy density Λ by applying the variation law for generalized Hubble's parameter that yields a constant value of deceleration parameter. We find that the constant value of deceleration parameter is reasonable for the present day universe. The variation law for Hubble's parameter generates two types of solutions for the average scale factor, one is of power-law type and other is of the exponential form. Using these two forms, Einstein's field equations are solved separately that correspond to expanding singular and non-singular models of the universe respectively. The cosmological constant Λ is found to be a decreasing function of time and positive which is corroborated by results from recent supernovae Ia observations. Expressions for look-back time-redshift, neoclassical tests (proper distance $d(z)$), luminosity distance red-shift and event horizon are derived and their significance are described in detail. The physical and geometric properties of spatially homogeneous and anisotropic cosmological models are discussed.

Keywords Bianchi type-V universe · Perfect fluid models · Decaying Λ · Heat conduction

1 Introduction

The study of Bianchi type V cosmological models create more interest as these models contain isotropic special cases and permit arbitrary small anisotropy levels at some instant of

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cosmic time. This property makes them suitable as model of our universe. The homogeneous and isotropic Friedman-Robertson-Walker (FRW) cosmological models, which are used to describe standard cosmological models, are particular case of Bianchi type I, V and IX universes, according to whether the constant curvature of the physical three-space, t = constant, is zero, negative or positive. These models will be interesting to construct cosmological models of the types which are of class one. Present cosmology is based on the FRW model which is completely homogeneous and isotropic. This is in agreement with observational data about the large scale structure of the universe. However, although homogeneous but anisotropic models are more restricted than the inhomogeneous models, they explain a number of observed phenomena quite satisfactorily. This stimulates the research for obtaining exact anisotropic solution for Einstein's field equations (EFEs) as a cosmologically accepted physical models for the universe (at least in the early stages). Among different models Bianchi type-V universes are the natural generalization of the open FRW model, which eventually become isotropic. Roy and Prasad [1] have investigated Bianchi type V universes which are locally rotationally symmetric and are of embedding class one filled with perfect fluid with heat conduction and radiation. Bianchi type V cosmological models have been studied by several researchers such as Farnsworth [2], Collins [3] Maartens and Nel [4], Wainwright et al. [5], Beesham [6], Maharaj and Beesham [7], Hewitt and Wainwright [8], Camci et al. [9], Meena and Bali [10], Pradhan et al. [11, 12], Aydogdu and Salti [13] in different physical contexts. Christodoulakis et al. [14–16] have studied un-tilted diffuse matter Bianchi V universe with perfect fluid and scalar field coupled to perfect fluid sources obeying a general equation of state. Following the work of Saha [17], Singh and Chaubey [18, 19] have obtained the quadrature form of metric function for Bianchi type-V model with perfect fluid and viscous fluid.

The effect of heat flow in the evolution of the universe has been investigated by several authors [20–27]. Banerjee and Sanyal [28] considered Bianchi type-V cosmological models with viscosity and heat flow. It has been shown that it is possible for dissipative Bianchi type-V cosmological models not to be in thermal equilibrium in their early stages. Coley [29] obtained Bianchi type-V spatially homogeneous imperfect fluid cosmological models in presence of both viscosity and heat flow. Coley and Hoogen [30] also generalized the work of Coley and Dunn [31] who considered a locally rotationally symmetric Bianchi type-V metric for an imperfect fluid source with both viscosity ans heat conduction. Recently Singh [32] and Singh et al. [33] have presented some new Bianchi type-V cosmological models in presence of perfect fluid with heat flow.

The problem of the cosmological constant is salient yet unsettled in cosmology. The smallness of the effective cosmological constant recently observed ($\Lambda_0 \leq 10^{-56}$ cm^{-2}) poses the most difficult problems involving cosmology and elementary particle physics theory. To explain the striking cancellation between the "bare" cosmological constant and the ordinary vacuum energy contributions of the quantum fields, many mechanisms have been proposed [34]. The "cosmological constant problem" can be expressed as the discrepancy between the negligible value of Λ for the present universe as seen by the successes of Newton's theory of gravitation [35] whereas the values 10^{50} larger is expected by the Glashow-Salam-Weinberg model [36] and by grand unified theory (GUT), it should be 10^{107} larger [37]. The cosmological term Λ is then small at the present epoch simply because the universe is too old. The problem in this approach is to determine the right dependence of Λ upon R or t.

Models with a relic cosmological constant Λ have received ample attention among researchers recently for various reasons (see Refs. [38–44] and references therein). Some of the recent discussions on the cosmological constant "problem" and on cosmology with a time-varying cosmological constant by Ratra and Peebles [45], Dolgov et al. [46, 47], Dolgov [48], and Sahni and Starobinsky [49] point out that in the absence of any interaction with matter or radiation, the cosmological constant remains a "constant", however, in the presence of interactions with matter or radiation, a solution of Einstein equations and the assumed equation of covariant conservation of stress-energy with a time-varying Λ can be found. For these solutions, conservation of energy requires decrease in the energy density of the vacuum component to be compensated by a corresponding increase in the energy density of matter or radiation. Earlier researches on this topic, are contained in Zeldovich [50], Weinberg [34] and Carroll, Press and Turner [51]. Recent observations by Perlmutter et al. [52–54] and Riess et al. [55, 56] strongly favor a significant and positive value of Λ. Their findings arising from the study of more than 50 type Ia supernovae with redshifts in the range $0.10 \le z \le 0.83$ suggest Friedmann models with negative pressure matter such as a cosmological constant (Λ), domain walls or cosmic strings (Vilenkin [57], Garnavich et al. [58, 59]). Recently, Carmeli and Kuzmenko [60], Behar and Carmeli [61] have shown that the cosmological relativistic theory predicts the value for cosmological constant $\Lambda = 1.934 \times 10^{-35}\ \mathrm{s}^{-2}$. This value of "$\Lambda$" is in excellent agreement with the measurements recently obtained by the High-Z Supernova Team and Supernova Cosmological Project (Garnavich et al. [58, 59], Perlmutter et al. [52–54], Riess et al. [55, 56], Schmidt et al. [62]). The main conclusion of these observations is that the expansion of the universe is accelerating.

Several ansätz have been proposed in which the Λ term decays with time (see Refs. Gasperini [63, 64], Berman [65–67], Berman et al. [68–70], Özer and Taha [40], Freese et al. [41, 42], Peebles and Ratra [71], Chen and Wu [72], Sattar and Viswakarma [73], Gariel and Le Denmat [74], Pradhan [75, 76], Pradhan et al. [77–85]). Of the special interest is the ansätz $\Lambda \propto S^{-2}$ (where S is the scale factor of the Robertson-Walker metric) by Chen and Wu [72], which has been considered or modified by several authors (Abdel-Rahaman [86, 87], Carvalho et al. [43], Silveira and Waga [44], Vishwakarma [88]).

Astronomical observations of large-scale distribution of galaxies of our universe show that the distribution of matter can be satisfactorily described by a perfect fluid. The adequacy of the isotropic universe for describing the present state of the universe is no basis for expecting that it is equally suitable for describing its early stages of evolution. Cosmological models which are spatially homogeneous but anisotropic have significant roles in the description of the universe at its early stages of evolution. Bianchi I–IX space-times are useful tools in constructing models of spatially homogeneous cosmologies (Ellis and MacCallum [89], Ryan and Shepley [90]). From these models, homogeneous Bianchi type V universes are the natural generalization of the open FRW model which eventually isotropize. Camci et al. [9] derived a new technique for generating exact solutions of EFEs with perfect fluid for Bianchi type V space-time. Recently Bali and Singh [91], Rao et al. [92–95], Tiwari [96], Singh and Baghel [97] and Singh et al. [33] have studied Bianchi type V cosmological models in different context.

Singh, Zeyauddin and Ram [33] have extended the work of Singh and Kumar [98, 99] to spatially homogeneous and totally anisotropic Bianchi type-V models with heat flow and perfect fluid as source. Zeyauddin and Ram [100] have recently obtained Bianchi type-V imperfect fluid cosmological models with heat flow. Recently, Tiwari [96] has obtained Bianchi type-V cosmological models with constant deceleration parameter within the framework of scalar-tensor theory of gravitation proposed by Saez and Ballester. In this paper, we propose to find Bianchi type V cosmological models in presence of perfect fluid distribution with heat flow and decaying vacuum energy density by using the law of variation for generalized mean Hubble's parameter and we will generalize the solutions of Ref. [33]. This paper is organized as follows: The metric and the law of variation for Hubble's parameter for Bianchi type-V model that yields constant value of the deceleration parameter is presented in Sect. 2. The field equations and the quadrature solutions of the metric functions in terms of average scale factor are presented in Sect. 3 for two types of cosmologies by using this law. In Sects. 3.1 and 3.2, exact solutions of the field equations are presented which correspond to the singular and non-singular cosmological models. These subsections also contain the geometric and physical properties of the derived models. Section 4 deals with the kinematic tests. Discussions and concluding remarks are given in Sect. 5.

2 The Metric and the Law of Variation for Hubble's Parameter

We consider the space time metric of the spatially homogeneous and anisotropic Bianchi type-V of the form

$$ds^2 = dt^2 - A^2dx^2 - e^{2mx}\left[B^2dy^2 + C^2dz^2\right], \tag{1}$$

where the metric potentials A, B and C are functions of cosmic time t alone and m is a constant.

We define the following physical and geometric parameters to be used in formulating the law and further in solving the Einstein's field equations for the metric (1).

The average scale factor a of Bianchi type-V model (1) is defined as

$$a = (ABC)^{\frac{1}{3}}. \tag{2}$$

A volume scale factor V is given by

$$V = a^3 = ABC. \tag{3}$$

We define the generalized mean Hubble's parameter H as

$$H = \frac{1}{3}(H_1 + H_2 + H_3), \tag{4}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble's parameters in the directions of x, y and z respectively. A dot stands for differentiation with respect to cosmic time t.

From (2)–(4), we obtain

$$H = \frac{1}{3}\frac{\dot{V}}{V} = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right). \tag{5}$$

Since the line element (1) is completely characterized by Hubble parameter H, therefore, let us consider that the mean Hubble parameter H is related to the average scale factor a by the relation

$$H = la^{-n} = l(ABC)^{-\frac{n}{3}}, \tag{6}$$

where l (> 0) and n (≥ 0) are constants. Such type of relations have already been considered by Berman [101], Berman and Gomide [102] for solving FRW models. Latter on many authors (see, Singh et al. [98, 99, 103], Singh and Baghel [97] and references therein) have studied flat FRW and Bianchi type models by using the special law for Hubble parameter that yields constant value of deceleration parameter.

An important observational quantity is the deceleration parameter q, which is defined as

$$q = -\frac{\ddot{a}a}{\dot{a}^2}. \tag{7}$$

From (5) and (6), we obtain

$$\dot{a} = la^{-n+1}, \tag{8}$$

$$\ddot{a} = -l^2(n-1)a^{-2n+1}. \tag{9}$$

Using (8) and (9) in to (7), we obtain

$$q = n - 1. \tag{10}$$

We observe that the relation (10) gives q as a constant. The sign of q indicated whether the model inflates or not. The positive sign of q i.e. ($q > 1$) correspond to "standard" decelerating model whereas the negative sign of q i.e. $q < 0$ indicates inflation. It is remarkable to mention here that though the current observations of SNe Ia and CMBR favors accelerating models ($q < 1$), but both do not altogether rule out the decelerating ones which are also consistent with these observations (see, Vishwakarma [88]).

From (8), we obtain the law for average scale factor a as

$$a = (nlt + c_1)^{\frac{1}{n}}, \tag{11}$$

for $n \neq 0$ and

$$a = c_2 \exp(lt), \tag{12}$$

for $n = 0$, where c_1 and c_2 are constants of integration. Equation (11) implies that the condition for the expansion of the universe is $n = q + 1 > 0$.

For $n \neq 0$, the age of the universe is given by

$$t_0 = \frac{1}{n}H_0^{-1} = \frac{1}{1+q}H_0^{-1}, \tag{13}$$

where subscript 0 stands for the present phase. For $n = 0$, the age of the universe is given by

$$t_0 = \ln\left(\frac{a_0}{c_2}\right)^{3/l}. \tag{14}$$

A numerical calculation can be made to estimate the present day age of the universe, the value of deceleration parameter compatible with the Supernovae observations. It is worth mentioned here that relations (11) and (12) independent on the particular gravitational theory being considered. It is approximately valid also for slowly time varying deceleration parameter. If $n > 0$, we expect that

$$\lim_{t\to\infty} a = \infty, \quad \lim_{a\to\infty} p = 0, \quad \lim_{a\to\infty} \rho = 0. \tag{15}$$

The physical quantities of observational interest in cosmology i.e. the expansion scalar θ, the average anisotropy parameter Am and the shear scalar σ^2 are defined as

$$\theta = u^i_{;i} = \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right), \tag{16}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left[\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2}\right] - \frac{\theta^2}{6}, \tag{17}$$

$$Am = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\triangle H_i}{H}\right)^2, \tag{18}$$

where $\triangle H_i = H_i - H$ $(i = 1, 2, 3)$.

3 Field Equations and Generation Technique

The influence of the perfect fluid with heat flow in the evolution of the universe is performed by means of its energy-momentum tensor, which acts as the source of the corresponding gravitational field. The energy-momentum tensor of a perfect fluid with heat conduction has the form [29–33]

$$T_{ij} = (p + \rho)u_i u_j - p g_{ij} + h_i u_j + h_j u_i, \tag{19}$$

where p is the thermodynamical pressure, ρ the energy density, u_i the four-velocity of the fluid and h_i is the heat flow vector satisfying

$$h^i u_i = 0. \tag{20}$$

In co-moving system of coordinates, we have $u_i = (0, 0, 0, 1)$.

The Einstein's field equations (in gravitational units $8\pi G = c = 1$) read as

$$R_{ij} - \frac{1}{2}R g_{ij} + \Lambda g_{ij} = -T_{ij}. \tag{21}$$

The field equations (21) and (20) imply that the heat flow is in x-direction only i.e. $h^i = (h_1(t), 0, 0, 0)$. For the energy momentum tensor (19) and Bianchi type V space-time (1), Einstein's field equations (21) yield the following five independent equations

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} = -p - \Lambda, \tag{22}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{m^2}{A^2} = -p - \Lambda, \tag{23}$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} = -p - \Lambda, \tag{24}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{3m^2}{A^2} = \rho - \Lambda, \tag{25}$$

$$m\left(\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = h_1. \tag{26}$$

The law of energy-conservation equation ($T^{ij}_{;j}=0$) gives

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=\frac{2m}{A^2}h_1-\dot{\Lambda}. \tag{27}$$

It is worth noting here that our approach suffers from a lack of Lagrangian approach. There is no known way to present a consistent Lagrangian model satisfying the necessary conditions discussed in this paper.

The field equations (22)–(25) and (27) can be reduced in terms of H, σ^2 and q as follows:

$$p+\Lambda=H^2(2q-1)-\sigma^2+\frac{m^2}{A^2}, \tag{28}$$

$$\rho-\Lambda=3H^2-\sigma^2-\frac{3m^2}{A^2}, \tag{29}$$

$$\dot{\rho}+3H(\rho+p)=\frac{2m}{A^2}h_1-\dot{\Lambda}. \tag{30}$$

We now describe the quadrature form of the Einstein's field equations (22)–(26). Here we follow the approach of Saha and Rikhvitsky [104], Singh and Chaubey [19] and Singh et al. [98, 99, 103] to solve the field equations. Subtracting (24) from (23), one finds the following relation between A and B

$$\frac{A}{B}=d_1\exp\left(k_1\int\frac{dt}{a^3}\right). \tag{31}$$

Analogically, we find the other following relations

$$\frac{A}{C}=d_2\exp\left(k_2\int\frac{dt}{a^3}\right), \tag{32}$$

and

$$\frac{B}{C}=d_3\exp\left(k_3\int\frac{dt}{a^3}\right), \tag{33}$$

where d_1, d_2, d_3, k_1, k_2 and k_3 are constants of integration. From (31)–(33), the metric functions can be written explicitly as

$$A(t)=\ell_1 a\exp\left(\frac{K_1}{3}\int\frac{dt}{a^3}\right), \tag{34}$$

$$B(t)=\ell_2 a\exp\left(\frac{K_2}{3}\int\frac{dt}{a^3}\right), \tag{35}$$

$$C(t)=\ell_3 a\exp\left(\frac{K_3}{3}\int\frac{dt}{a^3}\right), \tag{36}$$

where

$$K_1=k_1+k_2,\quad K_2=k_3-k_1,\quad K_3=-(k_2+k_3), \tag{37}$$

and

$$\ell_1=(d_1d_2)^{\frac{1}{3}},\quad \ell_2=(d_1^{-1}d_3)^{\frac{1}{3}},\quad \ell_3=(d_2d_3)^{-\frac{1}{3}}. \tag{38}$$

The constants K_1, K_2, K_3 and ℓ_1, ℓ_2, ℓ_3 satisfy the relations

$$K_1 + K_2 + K_3 = 0, \quad \text{and} \quad \ell_1\ell_2\ell_3 = 1. \tag{39}$$

Now we solve the exact solutions of quadrature equations (34)–(36) and corresponding pressure and energy density from (28) and (29) in the following two subsections depending on the values of n as defined in (11) and (12).

3.1 Solution with $n \neq 0$

Using the power-law form of the average scale factor $a(t)$, as given by (32), into (34)–(36), the solution for the metric functions can be written as

$$A(t) = \ell_1(nlt + c_1)^{\frac{1}{n}} \exp\left[\frac{K_1}{3l(n-3)}(nlt + c_1)^{\frac{(n-3)}{n}}\right], \tag{40}$$

$$B(t) = \ell_2(nlt + c_1)^{\frac{1}{n}} \exp\left[\frac{K_2}{3l(n-3)}(nlt + c_1)^{\frac{(n-3)}{n}}\right], \tag{41}$$

$$C(t) = \ell_3(nlt + c_1)^{\frac{1}{n}} \exp\left[\frac{K_3}{3l(n-3)}(nlt + c_1)^{\frac{(n-3)}{n}}\right], \tag{42}$$

where $n \neq 3$. Therefore the metric (1) reduces to

$$\begin{aligned} ds^2 = dt^2 &- \ell_1^2(nlt + c_1)^{\frac{2}{n}} \exp\left(\frac{2K_1}{3l(n-3)}(nlt + c_1)^{\frac{(n-3)}{n}}\right)dx^2 \\ &- e^{2mx}\left[\ell_2^2(nlt + c_1)^{\frac{2}{n}} \exp\left(\frac{2K_2}{3l(n-3)}(nlt + c_1)^{\frac{(n-3)}{n}}\right)dy^2\right. \\ &\left. + \ell_3^2(nlt + c_1)^{\frac{2}{n}} \exp\left(\frac{2K_3}{3l(n-3)}(nlt + c_1)^{\frac{(n-3)}{n}}\right)dz^2\right]. \end{aligned} \tag{43}$$

By using the transformation $x = X$, $y = Y$, $z = Z$, $nlt + c_1 = T$, the space-time (43) is reduced to

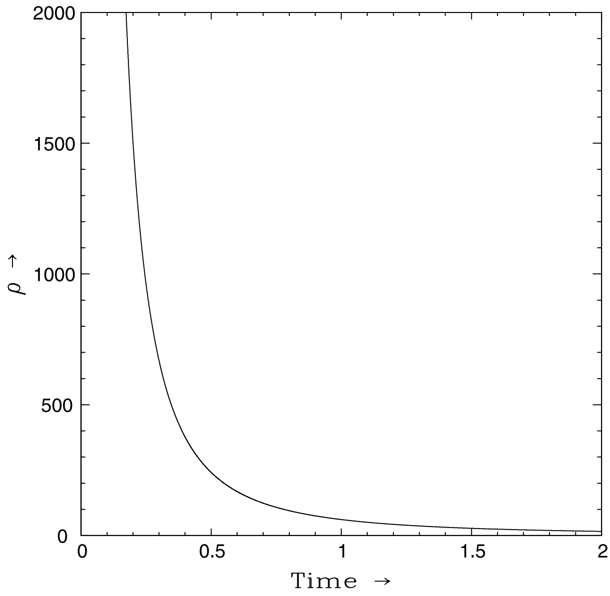
$$\begin{aligned} ds^2 = \frac{dT^2}{(nl)^2} &- \ell_1^2 T^{\frac{2}{n}} \exp\left(\frac{2K_1}{3l(n-3)}T^{\frac{(n-3)}{n}}\right)dX^2 \\ &- e^{2mx}\left[\ell_2^2 T^{\frac{2}{n}} \exp\left(\frac{2K_2}{3l(n-3)}T^{\frac{(n-3)}{n}}\right)dY^2\right. \\ &\left. + \ell_3^2 T^{\frac{2}{n}} \exp\left(\frac{2K_3}{3l(n-3)}T^{\frac{(n-3)}{n}}\right)dZ^2\right]. \end{aligned} \tag{44}$$

From (28) and (29), we compute the expressions for pressure p and density ρ for the model (44) and are given by

$$p = \frac{l^2(2n-3)}{T^2} - \frac{(K_1^2 + K_2^2 + K_3^2)}{18T^{\frac{6}{n}}} + \frac{m^2}{\ell_1^2 T^{\frac{2}{n}}} \exp\left[\frac{-2K_1 T^{\frac{(n-3)}{n}}}{3l(n-3)}\right] - \Lambda, \tag{45}$$

$$\rho = \frac{3l^2}{T^2} - \frac{(K_1^2 - K_2^2 + K_3^2)}{18T^{\frac{6}{n}}} - \frac{3m^2}{\ell_1^2 T^{\frac{2}{n}}} \exp\left[\frac{-2K_1 T^{\frac{(n-3)}{n}}}{3l(n-3)}\right] + \Lambda. \tag{46}$$

Fig. 1 The plot of energy density $\rho(t)$ vs. time



For the specification of $\Lambda(t)$, we assume that the fluid obeys an equation of state of the form

$$p = \gamma\rho, \tag{47}$$

where $\gamma (0 \leq \gamma \leq 1)$ is a constant.

Using (47) in (45) and (46), we obtain

$$(1+\gamma)\rho = \frac{2nl^2}{T^2} - \frac{(K_1^2+K_2^2+K_3^2)}{9T^{\frac{6}{n}}} - \frac{2m^2}{\ell_1^2 T^{\frac{2}{n}}} \exp\left[\frac{-2K_1 T^{\frac{(n-3)}{n}}}{3l(n-3)}\right]. \tag{48}$$

Eliminating $\rho(t)$ between (46) and (48), we obtain

$$(1+\gamma)\Lambda = -\frac{l^2(3\gamma-2n+3)}{T^2} + \frac{(1-\gamma)(K_1^2+K_2^2+K_3^2)}{18T^{\frac{6}{n}}} + \frac{m^2(3\gamma+1)}{\ell_1^2 T^{\frac{2}{n}}} \exp\left[\frac{-2K_1 T^{\frac{(n-3)}{n}}}{3l(n-3)}\right]. \tag{49}$$

From (26), we find

$$h_1 = \frac{mK_1}{T^{\frac{3}{n}}}. \tag{50}$$

Using above solutions, it can easily be seen that the energy conservation equation (27) is identically satisfied. Therefore, we have obtained exact solutions of Bianchi type-V cosmological model with perfect fluid and heat flow.

From (48), we observe that $\rho(t)$ is a decreasing function of time and $\rho > 0$ for all times. Figure 1 shows this behavior of energy density. From (49), we note the cosmological term Λ is a decreasing function of time and it approaches a small positive value with increase

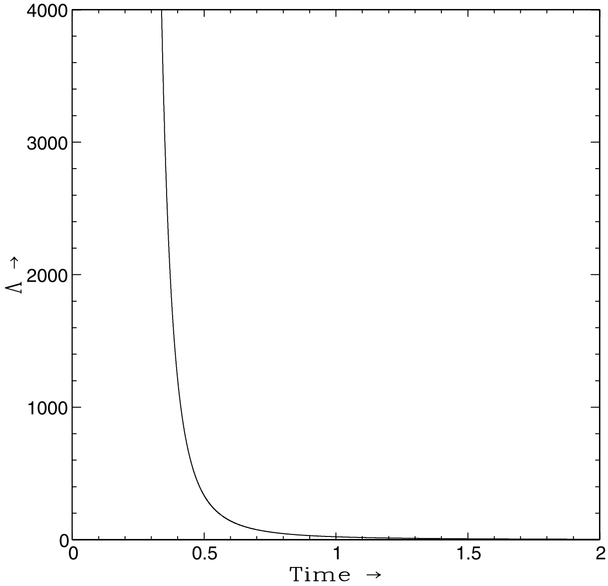


Fig. 2 The plot of cosmological term $\Lambda(t)$ vs. time

in time. From Fig. 2 (ρ and Λ are in geometrical units in entire paper), we note the same character of Λ. This is to be taken as a representative case of physical viability of the model.

The behavior of the universe in this model will be determined by the cosmological term Λ, this term has the same effect as a uniform mass density $\rho_{eff} = -\Lambda/4\pi$ which is constant in space and time. A positive value of Λ corresponds to a negative effective mass density (repulsion). Hence, we expect that in the universe with a positive value of Λ the expansion will tend to accelerate whereas in the universe with negative value of Λ the expansion will slow down, stop and reverse. In a universe with both matter and vacuum energy, there is a competition between the tendency of Λ to cause acceleration and the tendency of matter to cause deceleration with the ultimate fate of the universe depending on the precise amounts of each component. This continues to be true in the presence of spatial curvature, and with a nonzero cosmological constant it is no longer true that the negatively curved ("open") universes expand indefinitely while positively curved ("closed") universes will necessarily re-collapse—each of the four combinations of negative or positive curvature and eternal expansion or eventual re-collapse become possible for appropriate values of the parameters. There may even be a delicate balance, in which the competition between matter and vacuum energy is needed drawn and the universe is static (non expanding). The search for such a solution was Einstein's original motivation for introducing the cosmological constant. Recent cosmological observations (Garnavich et al. [58, 59], Perlmutter et al. [52–54], Riess et al. [55, 56], Schmidt et al. [62]) suggest the existence of a positive cosmological constant Λ with the magnitude $\Lambda(G\hbar/c^3) \approx 10^{-123}$. These observations on magnitude and red-shift of type Ia supernova suggest that our universe may be an accelerating one with induced cosmological density through the cosmological Λ-term. Thus the nature of Λ in our derived model of the universe is consistent with recent observations (Garnavich et al. [58, 59], Perlmutter et al. [52–54], Riess et al. [55, 56], Schmidt et al. [62]).

The expressions for kinematical parameters i.e. the scalar of expansion θ, shear scalar σ, average anisotropy parameter A_p and deceleration parameter q for the model (44) are given by

$$\theta = \frac{3l}{T}, \tag{51}$$

$$\sigma^2 = \frac{(K_1^2 + K_2^2 + K_3^2)}{18T^{-\frac{6}{n}}}, \tag{52}$$

$$A_p = \frac{(K_1^2 + K_2^2 + K_3^2)}{27l^2 T^{\frac{2(3-n)}{n}}}, \tag{53}$$

$$q = n - 1. \tag{54}$$

The rotation ω is identically zero.

The rate of expansion H_i in the direction of x, y and z are given by

$$H_x = \frac{l}{T} + \frac{K_1}{3T^{\frac{3}{n}}}, \tag{55}$$

$$H_y = \frac{l}{T} + \frac{K_2}{3T^{\frac{3}{n}}}, \tag{56}$$

$$H_z = \frac{l}{T} - \frac{K_3}{3T^{\frac{3}{n}}}. \tag{57}$$

Hence the average generalized Hubble's parameter is given by

$$H = \frac{l}{T}. \tag{58}$$

At the initial time $T = 0$, the physical parameters p, ρ and h_1 tend to infinity. Therefore, the universe starts evolving from initial singularity $T = 0$, with infinite internal pressure, infinite density and infinite internal heat flow. At initial stage of expansion p, ρ and Hubble parameter are large and θ decreases with the power-law type of expansion of the universe. The three scale factors are monotonically increasing function of time. The deceleration parameter is constant. We observe that the spatial scale factors are zero at the initial moment $T = 0$. The model has a point-type singularity. The heat conduction is decreasing function of time and is maximum at the initial epoch. The scale factors tend to infinity whereas p and ρ tend to zero as $T \to \infty$. The dynamics of the mean anisotropy parameter depends on the value of n. For $n < 3$, A_p has singular state, with infinite energy density and zero scale factors. For small time, A_p is increasing and in the large time limits, it ends in a homogeneous and isotropic state. Since $\lim_{T\to\infty} \frac{\sigma^2}{\theta^2} = 0$, the model approaches isotropy for large values of T. The heat conduction diminishes at $T \to \infty$. The anisotropic parameter A_p will end in a homogeneous and isotropic state for large time. All physical and kinematic parameters tend to zero as t tends to infinity.

3.2 Solution with $n = 0$

In this case we present an exponentially expanding non-singular cosmological model of the universe. Using the exponential form of the average scale factor $a(t)$, as given by (12), into the quadratures (34)–(36), we obtain the solution for the metric function

$$A(t) = c_2 \ell_1 \exp\left[lt - \frac{K_1}{9lc_2^3} \exp(-3lt)\right], \tag{59}$$

$$B(t) = c_2\ell_2 \exp\left[lt - \frac{K_2}{9lc_2^3}\exp(-3lt)\right], \tag{60}$$

$$C(t) = c_2\ell_3 \exp\left[lt - \frac{K_3}{9lc_2^3}\exp(-3lt)\right]. \tag{61}$$

Therefore the metric (1) reduces to

$$\begin{aligned} ds^2 = dt^2 &- c_2^2\ell_1^2 \exp\left(2lt - \frac{2K_1}{9lc_2^3}\exp(-3lt)\right)dx^2 \\ &- e^{2mx}\left[c_2^2\ell_2^2 \exp\left(2lt - \frac{2K_2}{9lc_2^3}\exp(-3lt)\right)dy^2\right. \\ &\left.+ c_2^2\ell_3^2 \exp\left(2lt - \frac{2K_1}{9lc_2^3}\exp(-3lt)\right)dz^2\right]. \end{aligned} \tag{62}$$

From (28) and (29), we compute the expressions for pressure p and density ρ for the model (62) and are given by

$$\begin{aligned} p = -3l^2 &- \frac{(K_1^2 + K_2^2 + K_3^2)}{9c_2^6}\exp(-6lt) \\ &+ \frac{m^2}{c_2^2\ell_1^2}\exp\left[-2\left(lt - \frac{K_1}{9lc_2^3}\exp(-3lt)\right)\right] - \Lambda, \end{aligned} \tag{63}$$

$$\begin{aligned} \rho = 3l^2 &+ \frac{2l^2(K_1^2 + K_2^2 + K_3^2)}{3c_2^3}\exp(-3lt) \\ &+ \frac{(K_1K_2 + K_1K_3 + K_2K_3)}{9c_2^6}\exp(-6lt) \\ &- \frac{3m^2}{c_2^2\ell_1^2}\exp\left[-2\left(lt - \frac{K_1}{9lc_2^3}\exp(-3lt)\right)\right] + \Lambda. \end{aligned} \tag{64}$$

For the specification of $\Lambda(t)$, we use the equation of state (47) in (63) and (64), we obtain

$$\begin{aligned} (1+\gamma)\rho = &\frac{2l(K_1 + K_2 + K_3)}{3c_2^3}\exp(-3lt) \\ &+ \frac{(-K_1^2 - K_2^2 + K_1K_3 + K_2K_3)}{9c_2^6}\exp(-6lt) \\ &- \frac{2m^2}{c_2^2\ell_1^2}\exp\left[-2\left(lt - \frac{K_1}{9lc_2^3}\exp(-3lt)\right)\right]. \end{aligned} \tag{65}$$

Eliminating $\rho(t)$ between (64) and (65), we obtain

$$\begin{aligned} (1+\gamma)\Lambda = -3(1+\gamma)l^2 &- \frac{2\gamma l(K_1 + K_2 + K_3)}{3c_2^3}\exp(-3lt) \\ &- \frac{\gamma(K_1K_2 + K_1K_3 + K_2K_3 + K_1^2 + K_2^2 + K_3^2)}{9c_2^6}\exp(-6lt) \end{aligned}$$

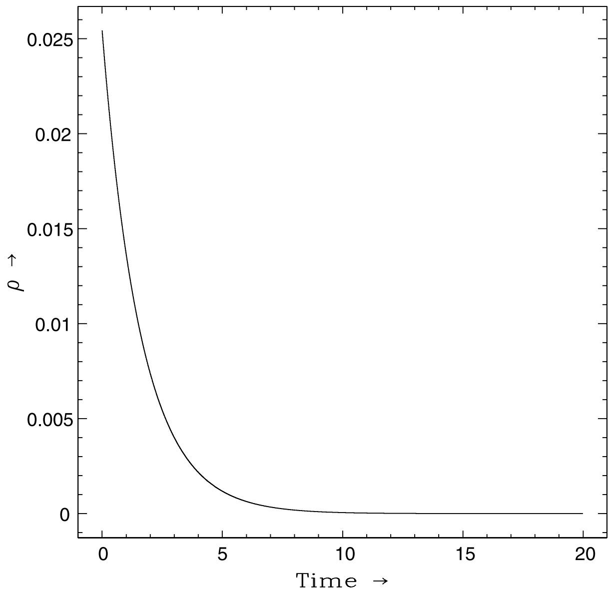


Fig. 3 The plot of energy density $\rho(t)$ vs. time

$$-\frac{(3\gamma+1)m^2}{c_2^2\ell_1^2}\exp\left[-2\left(lt-\frac{K_1}{9lc_2^2}\exp(-3lt)\right)\right]. \tag{66}$$

From (26), we find

$$h_1=\frac{mK_1}{c_2^3}\exp(-3lt). \tag{67}$$

Using above solutions, it can easily be seen that the energy conservation equation (27) is identically satisfied.

From (65), we observe that $\rho(t)$ is a decreasing function of time and $\rho > 0$ for all times. Figure 3 shows this behavior of energy density. From (66), we note the cosmological term Λ is a decreasing function of time and it approaches a small positive value with increase in time. Figure 4 shows this behavior Λ. Thus the nature of Λ in our derived model of the universe is consistent with recent observations (Garnavich et al. [58, 59], Perlmutter et al. [52–54], Riess et al. [55, 56], Schmidt et al. [62]).

The expressions for kinematical parameters i.e. the scalar of expansion θ, shear scalar σ, average anisotropy parameter A_p and deceleration parameter q for the model (62) are given by

$$\theta=3l, \tag{68}$$

$$\sigma^2=\frac{(K_1^2+K_2^2+K_3^2)}{18c_2^6}\exp(-6lt), \tag{69}$$

$$A_p=\frac{(K_1^2+K_2^2+K_3^2)}{27l^2c_2^6}\exp(-6lt), \tag{70}$$

$$q=-1. \tag{71}$$

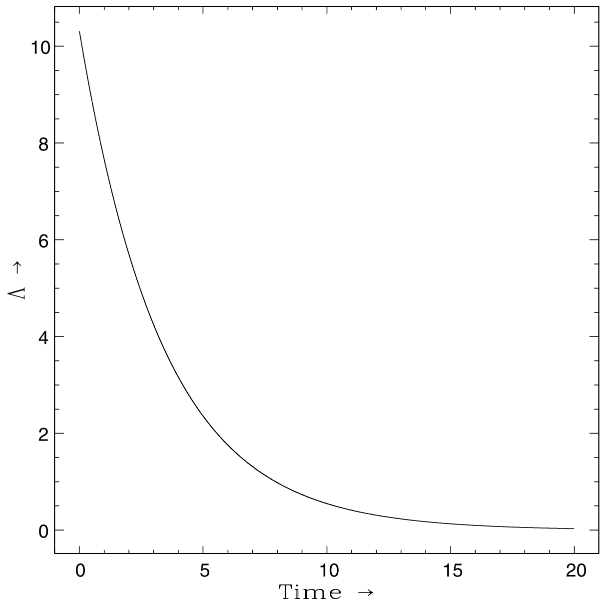


Fig. 4 The plot of cosmological term $\Lambda(t)$ vs. time

The rotation ω is identically zero. The negative value of q indicates inflation. The evolution of the universe in such a scenario is consistent with the present day observations predicting an accelerated expansion.

The rate of expansion H_i in the direction of x, y and z are given by

$$H_x = l + \frac{K_1}{3c_2^3} \exp(-3lt), \tag{72}$$

$$H_y = l + \frac{K_2}{3c_2^3} \exp(-3lt), \tag{73}$$

$$H_z = l + \frac{K_3}{3c_2^3} \exp(-3lt). \tag{74}$$

Hence the average generalized Hubble's parameter is given by

$$H = l. \tag{75}$$

From (72)–(75), we observe that the directional Hubble parameters are time dependent while the average Hubble parameter is constant.

It is observed that the physical and kinematic quantities are all constant at $t = 0$. The kinematic parameters tend to zero as $t \to \infty$. The heat conduction is decreasing function of time and is constant at $t = 0$. The heat flow diminishes for large time. This shows that the universe starts evolving with constant volume and expands exponentially. The expansion in the model is uniform throughout the time of evolution. We find that $\lim_{t\to\infty} \frac{\sigma^2}{\theta^2} = 0$, which shows that this inflationary universe eventually approaches isotropy for large values of t. The derived model is non-singular. If we set $\Lambda = 0$ in our solutions, we get the solutions obtained by Singh et al. [33].

4 Kinematical Tests

The values of $a(t)$ derived in (11) and (12) may be used in formulating the kinematics tests for any arbitrary large red-shifts. We now study the consistency of the derived models for both cases with the observational parameters through kinematics tests.

4.1 When $n \neq 0$

4.1.1 Look Back Time

The time in the past at which the light we now receive from a distant object was emitted is called the look back time. How *long ago* the light was emitted (the look back time) depends on the dynamics of the universe.

The radiation travel time (or look-back time) $T - T_0$ for photon emitted by a source at instant T and received at T_0 is given by

$$T - T_0 = \int_a^{a_0} \frac{da}{\dot{a}}. \tag{76}$$

Equation (11) can be rewritten as

$$a = T^{\frac{1}{n}}. \tag{77}$$

This follows that

$$\frac{a_0}{a} = 1 + z = \left(\frac{T_0}{T}\right)^{\frac{1}{n}}, \tag{78}$$

where a_0 is the present scale factor. The above equation gives

$$T = T_0(1+z)^{-n}. \tag{79}$$

From (13) and (79), we obtain

$$T_0 - T = \frac{H_0^{-1}}{n}\left[1 - (1+z)^{-n}\right], \tag{80}$$

which is

$$H_0(T_0 - T) = \frac{1}{n}\left[1 - (1+z)^{-n}\right], \tag{81}$$

where H_0 is Hubble's constant at present in $\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and its value is believed to be somewhere between 50 and 100 $\mathrm{km\,s^{-1}\,Mpc^{-1}}$. However, the reciprocal of Hubble's constant is called the Hubble time T_H, i.e., $T_H = H_0^{-1}$, where T_H is expressed in s and H_0 in s^{-1}. For small z one obtain

$$H_0(T_0 - T) = \frac{1}{n}\left[nz - \frac{n(n-1)}{2}z^2 + \cdots\right]. \tag{82}$$

The above relation can be transformed by using $q = (n-1)$ into

$$H_0(T_0 - T) = z - \frac{q}{2}z^2 + \cdots. \tag{83}$$

Taking limit $z \to \infty$ in (81), the present age of the universe (the extrapolated time back to the bang) is

$$T_0 = \frac{H_0^{-1}}{n} = \frac{H_0^{-1}}{1+q}, \tag{84}$$

which is same as expected in (13). Putting $n = \frac{3}{2}$ in (84), we obtain the result of well-known Einstein-de-Sitter universe

$$H_0(T_0 - T) = \frac{2}{3}\left[1 - (1+z)^{-\frac{3}{2}}\right]. \tag{85}$$

It is remarkable to mention here that relation (85) is used to describe look-back time in Einstein-de-Sitter universe. In the limit $z \to \infty$, (85) is reduced to

$$T_0 = \frac{2}{3} H_0^{-1} = \frac{2}{3} T_H. \tag{86}$$

4.1.2 Neoclassical Tests (Proper Distance $d(z)$)

A photon emitted by a source with coordinate $r = r_1$ and $T = T_1$ and received at a time T_0 by an observer located at $r = 0$. The emitted radiation will follow null geodesics on which $(\theta_1, \theta_2, \ldots, \theta_n)$ are constant.

The proper distance between the source and observer is given by

$$d(z) = a_0 \int_a^{a_0} \frac{da}{a\dot{a}}, \tag{87}$$

$$r_1 = \int_{T_1}^{T_0} \frac{dT}{a} = \frac{a_0^{-1} H_0^{-1}}{(n-1)} \left[1 - (1+z)^{1-n}\right].$$

Hence

$$d(z) = r_1 a_0 = \frac{H_0^{-1}}{n-1}\left[1 - (1+z)^{1-n}\right], \tag{88}$$

where $(1+z) = \frac{R_0}{R}$ = red-shift and a_0 is the present scale factor of the universe.

For small z, (88) reduces to

$$H_0 d(z) = z - \frac{1}{2n} z^2 + \cdots. \tag{89}$$

By using (13) in above equation, we obtain

$$H_0 d(z) = z - \frac{1}{2}(1+q) z^2 + \cdots. \tag{90}$$

From (88), it is observed that the distance d is maximum at $z = \infty$. Hence

$$d(z = \infty) = H_0^{-1}\left(\frac{1}{n-1}\right). \tag{91}$$

Equation (88) gives the Freese et al. [41, 42] results for the proper distance if we choose $n = 2$.

4.1.3 Luminosity Distance

Luminosity distance is the another important concept of theoretical cosmology of a light source. The luminosity distance is a way of expanding the amount of light received from a distant object. It is the distance that the object appears to have, assuming the inverse square law for the reduction of light intensity with distance holds. The luminosity distance is *not* the actual distance to the object, because in the real universe the inverse square law does not hold. It is broken both because the geometry of the universe need not be flat, and because the universe is expanding. In other words, it is defined in such a way as generalizes the inverse-square law of the brightness in the static Euclidean space to an expanding curved space [105].

If d_L is the luminosity distance to the object, then

$$d_L = \left(\frac{L}{4\pi l}\right)^{\frac{1}{2}}, \tag{92}$$

where L is the total energy emitted by the source per unit time, l is the apparent luminosity of the object. Therefore one can write

$$d_L = r_1 a_0 = d(1+z). \tag{93}$$

Using (88) in (93) reduces to

$$H_0 d_L = \frac{(1+z)}{n-1}\left[1-(1+z)^{1-n}\right]. \tag{94}$$

For small value of z, (94) gives

$$H_0 d_L = z + \frac{1}{2}(1-q)z^2 + \cdots . \tag{95}$$

The luminosity distance depends on the cosmological model we have under discussion, and hence can be used to tell us which cosmological model describe our universe. Unfortunately, however, the observable quantity is the radiation flux density received from an object, and this can only be translated into a luminosity distance if the absolute luminosity of the object is known. There is no distant astronomical objects for which this is the cases. This problem can however be circumvented if there are a population of objects at different distances which are believed to have the same luminosity; even if that luminosity is not known, it will appear merely as an overall scaling factor.

4.2 When $n = 0$

4.2.1 Look Back Time

In this case we obtain

$$H_0(\tau_0 - \tau) = \log(1+z). \tag{96}$$

For small z, we have

$$H_0(\tau_0 - \tau) = \left[z - \frac{z^2}{2} + \cdots .\right]. \tag{97}$$

4.2.2 Luminosity Distance

Here we obtain

$$d_L = \frac{1}{H_0}(z + z^2), \tag{98}$$

which shows that the luminosity distance increases faster with red-shift z for $q = -1$.

4.2.3 Event Horizon

The solution exists an event horizon which is given by

$$r_E = a(t_0)\int_{\tau_0}^{\infty} \frac{d\tau}{a(\tau)} = \frac{1}{c_2 H}. \tag{99}$$

This value of the limit gives the event horizon where no observer beyond a proper distance r_E at $\tau = \tau_0$ can communicate with another observer.

5 Discussion and Concluding Remarks

In this paper we have generalized the solutions obtained by Singh, Zeyauddin and Ram [33]. The proposal of a law of variation for Hubble's parameter that yields a constant value of deceleration parameter is discussed in homogeneous and anisotropic Bianchi type-V space-time in general relativity. The law of variation for Hubble's parameter defined in (6) for Bianchi type-V space-time model gives two types of cosmologies, (i) first form (for $n \neq 0$) shows the solution for positive value of deceleration parameter indicating the power law expansion of the universe whereas (ii) second one (for $n = 0$) shows the solution for negative value of deceleration parameter, which shows the exponential expansion of the universe. Exact solutions of Einstein's field equations for this model of the universe have been obtained by using the two forms of average scale factor. The power law solutions represent the singular model where the spatial scale factors and volume scalar vanish at $T = 0$. The energy density and pressure are infinite at this initial epoch. As $T \to \infty$, the scale factors diverge and p, ρ both tend to zero. A_m and σ^2 are very large at initial time but decrease with cosmic time and vanish as $t \to \infty$. The model shows isotropic state in later time of its evolution. The exponential solutions represent singularity free model of the universe. In this case as $T \to -\infty$, the scale factors tend to zero which indicates that the universe is infinitely old and has exponential inflationary phase. All the parameters such as scale factor, p, ρ, θ, σ^2 and A_m are constant at $T = 0$. The rate of expansion of the universe is uniform through out the evolution.

Under the law of variation for Hubble's parameter defined in (6), it has been shown that the two classes of solutions lead to the conclusion that, if $q > 0$ the model expands but always decelerate whereas $q < 0$ provides the exponential expansion and later accelerates the universe. The evolution of the universe in such a scenario is shown to be consistent with the present observations predicting an accelerated expansion. We have also described the well-known astrophysical phenomena, namely look-back time, neoclassical tests, luminosity distance and event horizon with red-shift. It has been observed that such models of the universe are compatible with present observations. It is observed that luminosity distance increases linearly with red-shift for $q = 1$ whereas it increases faster with red-shift z for $q = 0$ and $q = -1$. The solutions obtained in the present paper could give an appropriate description of the evolution of universe. In summary, we have extended the law of variation of Hubble parameter proposed by Berman [101] to Bianchi type V space-time to investigate the exact solutions of Einstein's field equations.

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